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Microeconomics of the Russian Family Support System

"*No Woman, No Cry*"
Bob Marley

1 Introduction

The goal of this essay is to theoretically assess the family support system in Russia. The focus will be held on the maternity (family) capital and its influence on family consumption and utility. The research question has been inspired by the paper by Andreeva, Bychkov, and Feoktistova (ISSN 2410-1613).

Apparently, the government imposing such a support policy assumes that it will solve two problems at once: the birthrate will increase, while the poverty rate will decrease. In reality, it is not the case: the birthrate increases, however, the total experienced utility falls due to the fall in consumption which means that poverty only increases.

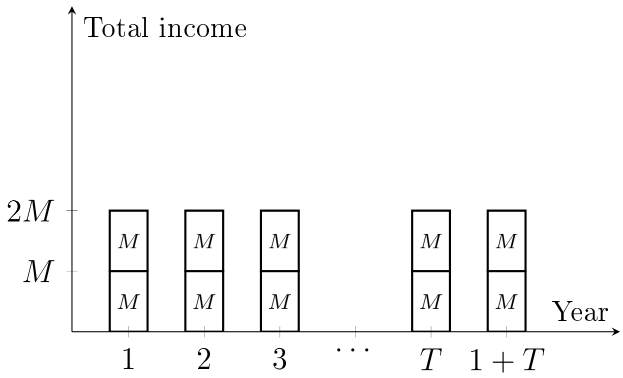
The model is designed to assess the problem applying microeconomic approach.

2 Assumptions

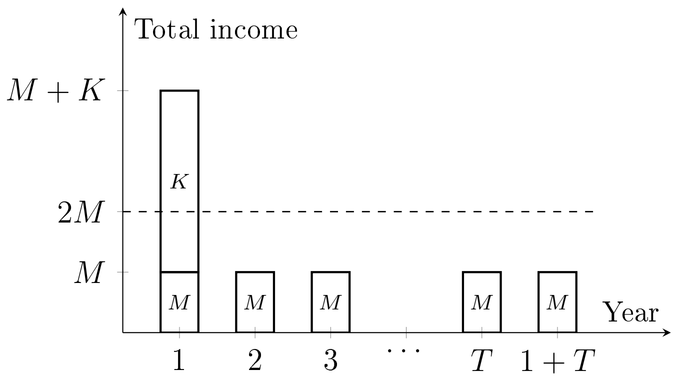
To deliver the model the following assumptions should be introduced:

1. To assess the successfulness of the policy implemented by the government, a household of a man and a woman is analysed. Both partners earn the same wage rate M and have no savings accumulated. In the beginning they live in a rented apartment. At the start of year 1 the family makes a choice between the two alternative scenarios:
   - Scenario one (further on it will be referred as "no K" case): the family decides not to have a child and stay at the rented flat with no maternity capital.
   - Scenario two (further on it will be referred as "K" case): the family decides to have a child and therefore to get the maternity capital which they fully spend in year 1 as the down payment to buy a mortgage apartment, but starting from year 2 during T years they pay back the remaining part of the mortgage from their own income. For simplicity, it is assumed that mother is on maternity leave forever.
2. The family spends income only on goods consumption and flat (rent or mortgage payments). Preferences of the partners are identical. They are presented by the logarithmised Cobb-Douglas utility function with real consumption ($C/\mathbb{P}$) and apartment area per adult (F/n) as arguments because of its reasonableness and mathematically convenient properties.
3. The utility discount factor (δ) is introduced to reflect the fact that an adult values utility in year t more that in year $t+1$.
4. The price level in the neighborhood ($\mathbb{P}$) is required to underline the difference between different prices in cities and to make a transition to real consumption.
5. The apartment price per square meter (p) incorporates the bank interest (for simplicity).
6. Additionally, one technical assumption is introduced to simplify calculations. It is discussed in section 4.2 of the paper.

Histograms below illustrate how annual income of the family behaves under each scenario.



Family income profile
in the "no K" case



Family income profile
in the "K" case

3 Notations

- M — adult's income, $M > 0$
- $\mathbb{P}$ — price level in the neighborhood, $\mathbb{P} > 0$
- p — apartment price per square meter, $p > 0$
- r — annual rent per square meter, $r > 0$
- β — time-averaged newborn's consumption as a share from the adult's consumption, $\beta > 0$
- γ — consumption weight in the Cobb-Douglas utility function, $0 < \gamma < 1$
- δ — utility discount factor, $0 < \delta < 1$
- K — maternity capital value, $K > 0$
- T — mortgage tenure in years equal to the number of installments excluding the down payment made in year 1, $T > 0$
- C — annual consumption per adult (nominal), $C > 0$
- $C/\mathbb{P}$ — annual consumption per adult (real), $C/\mathbb{P} > 0$
- F — apartment area (regardless property rights), $F > 0$

4 Model Setup

Utility received by an adult during one year is described by the following function:

$$u\left(\frac{C}{\mathbb{P}},\frac{F}{n}\right) = \gamma \cdot ln\left(\frac{C}{\mathbb{P}}\right) + (1-\gamma)\cdot ln\left(\frac{F}{n}\right),$$

where n is the number of consumers in a family expressed in adult equivalents:

$$n = \begin{cases} 2, & \text{"no K" case} \\ 2+\beta, & \text{"K" case} \end{cases}.$$

The family decides whether to give birth to a child and to receive maternity capital or not. To choose between these two options, they solve two utility maximization problems (UMPs): the first one corresponds to the "no K" case (no child — no maternity capital), whereas the second represents the "K" case (one child — maternity capital).

Adults maximize their utility function TU_{noK} with respect to C_0 and F_0 in the "no K" case and the function TU_K with respect to C_2 and F_1 in the "K" case. Then optimal pairs (C^*,F^*) are plugged back to the corresponding total utility functions, and the resulting optimized total utility figures TU^*_{noK} and TU^*_K are compared. Finally, the case resulting in a larger value of total utility is chosen. Further on TU functions will be referred as decision utility functions.

4.1 "No K" case

In the "no K" case (no children — no maternity capital) every year each adult receives fixed utility

$$u_0 = u\left(\frac{C_0}{\mathbb{P}},\frac{F_0}{2}\right) = \gamma \cdot ln\left(\frac{C_0}{\mathbb{P}}\right) + (1-\gamma)\cdot ln\left(\frac{F_0}{2}\right).$$

Implementing discounting we arrive to the total decision utility function given by

$$TU_{noK} = u_0 + \delta u_0 + \delta^2 u_0 + \dots = (1+\delta+\delta^2+\dots)u_0 = \frac{1}{1-\delta}u_0.$$

Budget constraint in the "no K" case in the same each year. It is derived as follows:

$$2M \geq 2C_0 + rF_0 \quad \Rightarrow \quad 2M = 2C_0 + rF_0 \quad \Leftrightarrow \quad C_0 = \frac{2M - rF_0}{2}.$$

The constraint is binding as otherwise it is possible to rise C_0 and F_0 a bit under the same rent level r, so that the inequality is satisfied but u_0 and therefore TU_{noK} increases.
Thus, UMP in the "no K" case is given by:

$$\begin{cases} TU_{noK} \to \max\limits_{C_0,F_0\geq 0} \\ \text{s.t.} \quad C_0 = \frac{2M-rF_0}{2} \end{cases} \Leftrightarrow \begin{cases} \frac{1}{1-\delta}u_0 \to \max\limits_{C_0,F_0\geq 0} \\ \text{s.t.} \quad C_0 = \frac{2M-rF_0}{2} \end{cases} \Leftrightarrow \begin{cases} u_0 \to \max\limits_{C_0,F_0\geq 0} \\ \text{s.t.} \quad C_0 = \frac{2M-rF_0}{2} \end{cases}$$

4.2 "K" case

In the "K" case (one child — maternity capital) utility u_1 received by an adult in year 1 differs from the annual utility u_2 he/she receives starting from year 2. The reason is that consumption in year 1 (C_1) exceeds that in each of the subsequent years (C_2) since in year 1 the family pays for mortgage from the maternity capital, not from their own earnings:

$$u_1 = u\left(\frac{C_1}{\mathbb{P}}, \frac{F_1}{2+\beta}\right) = \gamma \cdot ln\left(\frac{C_1}{\mathbb{P}}\right) + (1-\gamma) \cdot ln\left(\frac{F_1}{2+\beta}\right)$$
$$u_2 = u\left(\frac{C_2}{\mathbb{P}}, \frac{F_1}{2+\beta}\right) = \gamma \cdot ln\left(\frac{C_2}{\mathbb{P}}\right) + (1-\gamma) \cdot ln\left(\frac{F_1}{2+\beta}\right)$$

Decision utility function in the "K" case is given by

$$TU_K = u_1 + \delta u_2 + \delta^2 u_2 + \delta^3 u_2 + \dots = u_1 + (\delta + \delta^2 + \delta^3 + \dots)u_2 = u_1 + \frac{\delta}{1-\delta} u_2.$$

At this stage a simplifying assumption was used. Since the installment happening in year $1+T$ finalizes the mortgage repayment, then starting from the next year consumption per person will rise. Hence, in the long run annual utility received by a family member exceeds u_2. However, to simplify calculations the decision utility function TU_K is computed *as if annual utility takes the value of u_2 from year 2 forever*. The expression above underestimates the TU_K function, but as delayed utilities are discounted almost completely then *for all reasonable values of δ and T* the difference between actual TU_K and the proposed approximation can be neglected.

After giving birth the mother stops working and as a result annual income of the family is earned by husband only and equals M. Note that in year 1 all this income is spent on consumption of adults and the baby:

$$C_1 = \frac{M}{2+\beta} = const.$$

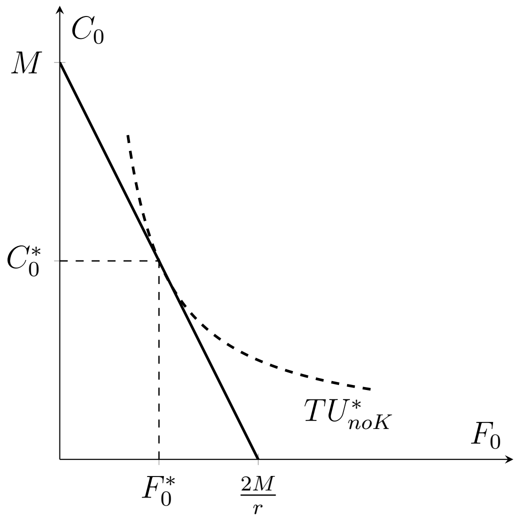
In years from 2 to $1+T$ income is spent on consumption of the family members and mortgage payments. Thus, the budget constraint in the "K" case takes form

$$M \geq (2+\beta)C_2 + \frac{pF_1 - K}{T} \quad \Rightarrow \quad M = (2+\beta)C_2 + \frac{pF_1 - K}{T} \quad \Leftrightarrow$$
$$\Leftrightarrow \quad C_2 = \frac{M - \frac{pF_1 - K}{T}}{2+\beta} \quad \Leftrightarrow \quad C_2 = \frac{(MT+K) - pF_1}{(2+\beta)T}.$$

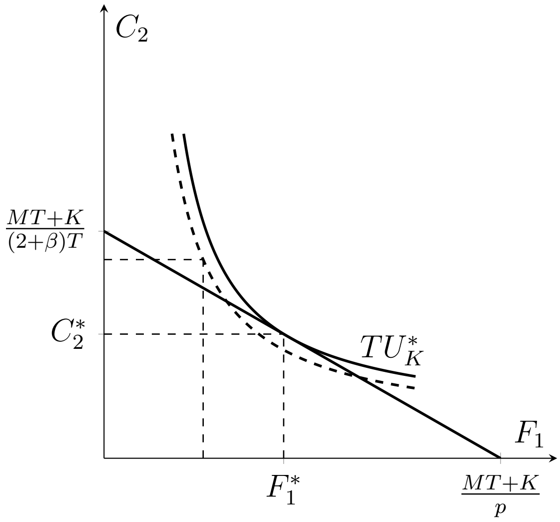
The constraint is binding as otherwise it is possible to rise C_2 and F_1 a bit under the same apartment price p, so that the inequality is satisfied but u_1 and u_2 increase, and therefore TU_K also increases.
So, UMP in the "K" case is given by:

$$\begin{cases} TU_K \to \max\limits_{C_2, F_1 \geq 0} \\ \text{s.t.} \quad C_1 = \frac{M}{2+\beta} \\ \qquad\; C_2 = \frac{(MT+K)-pF_1}{(2+\beta)T} \end{cases} \Leftrightarrow \begin{cases} u_1 + \frac{\delta}{1-\delta} u_2 \to \max\limits_{C_2, F_1 \geq 0} \\ \text{s.t.} \quad C_1 = \frac{M}{2+\beta} \\ \qquad\; C_2 = \frac{(MT+K)-pF_1}{(2+\beta)T} \end{cases}$$

Graphs below illustrate UMPs' solutions for the situation when family chooses to give birth ($TU^*_{noK} < TU^*_K$). Maternity capital allows them to increase apartment area ($F^*_0 < F^*_1$), but starting from year 2 nominal consumption per adult falls ($C^*_0 > C^*_2$).



Choice in the "no K" case



Choice in the "K" case

4.3 Aims of the Model

The aims of the model are:

1. To solve the two outlined UMPs and to find the optimal C^* and F^* in each case.
2. To find the minimum level of maternity capital K_{min} under which the family chooses the "K" case (to give birth) over the "no K" case (further on the value K_{min} will be refered as separating capital level).
3. To analyze the effects of different variables (especially, income) on K_{min}.
4. To demonstrate that the give-birth "K" case leads to a decrease in welfare (calculated as non-discounted sum of experienced utilities).

These aims are accomplished one by one in the next part of the paper.

5 Derivation

5.1 Solution of UMPs

"No K" case

UMP in the "no K" case results in the following expressions for maximizers:

$$\begin{cases} u\left(\frac{C_0}{\mathbb{P}}, \frac{F_0}{2}\right) \to \max\limits_{C_0, F_0 \geq 0} \\ \text{s.t.} \quad C_0 = \frac{2M - rF_0}{2} \end{cases} \Rightarrow \quad \begin{cases} C^*_0 = \gamma M \\ F^*_0 = \frac{(1-\gamma)\cdot 2M}{r} \end{cases}$$

Plugging C_0^* and F_0^* back into $u_0 = u\left(\frac{C_0}{\mathbb{P}}, \frac{F_0}{2}\right)$ we find the indirect utility function

$$\begin{aligned} TU_{noK}^* &= \frac{1}{1-\delta} \cdot u_0^* = \\ &= \frac{1}{1-\delta} \cdot u\left(\frac{C_0^*}{\mathbb{P}}, \frac{F_0^*}{2}\right) = \\ &= \frac{1}{1-\delta}\left[\gamma \cdot ln\left(\frac{\gamma M}{\mathbb{P}}\right) + (1-\gamma) \cdot ln\left(\frac{(1-\gamma)M}{r}\right)\right]. \end{aligned}$$

"K" case

UMP in the "K" case gives the following results:

$$\begin{cases} u\left(\frac{C_1}{\mathbb{P}}, \frac{F_1}{2+\beta}\right) + \frac{\delta}{1-\delta} u\left(\frac{C_2}{\mathbb{P}}, \frac{F_1}{2+\beta}\right) \to \max\limits_{C_2, F_1 \geq 0} \\ \text{s.t.} \quad C_1 = \frac{M}{2+\beta} \\ \qquad C_2 = \frac{(MT+K)-pF_1}{(2+\beta)T} \end{cases} \Rightarrow \begin{cases} C_2^* = (1-A) \times \frac{MT+K}{(2+\beta)T} \\ F_1^* = A \times \frac{MT+K}{p} \end{cases},$$

where $A = \frac{1-\gamma}{1-\gamma+\delta\gamma}$ and thus $1 - A = \frac{\delta\gamma}{1-\gamma+\delta\gamma}$.
Plugging $C_1 = \frac{M}{2+\beta}$, C_2^* and F_1^* back into $u_1 = u\left(\frac{C_1}{\mathbb{P}}, \frac{F_1}{2+\beta}\right)$ and $u_2 = u\left(\frac{C_2}{\mathbb{P}}, \frac{F_1}{2+\beta}\right)$ we find the indirect utility function

$$\begin{aligned} TU_K^*(K) &= u_1^* + \frac{\delta}{1-\delta} \cdot u_2^* = \\ &= u\left(\frac{C_1}{\mathbb{P}}, \frac{F_1^*}{2+\beta}\right) + \frac{\delta}{1-\delta} u\left(\frac{C_2^*}{\mathbb{P}}, \frac{F_1^*}{2+\beta}\right) = \\ &= \left[\gamma \cdot ln\left(\frac{M}{(2+\beta)\mathbb{P}}\right) + (1-\gamma) \cdot ln\left(\frac{A(MT+K)}{(2+\beta)p}\right)\right] + \\ &+ \frac{\delta}{1-\delta}\left[\gamma \cdot ln\left(\frac{(1-A)(MT+K)}{(2+\beta)T\mathbb{P}}\right) + (1-\gamma) \cdot ln\left(\frac{A(MT+K)}{(2+\beta)p}\right)\right]. \end{aligned}$$

5.2 Calculation of the Separating Capital Level

The separating capital level K_{min} is the minimal value of K under which the couple chooses to give birth to a child, i.e. such that the following inequality holds:

$$TU_K^*(K) \geq TU_{noK}^*.$$

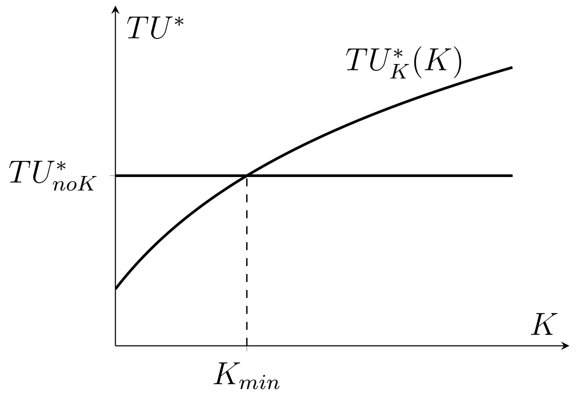
Therefore, K_{min} is the solution of equation

$$TU_K^*(K_{min}) = TU_{noK}^*.$$

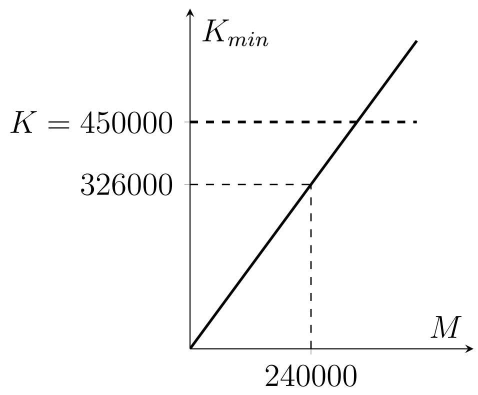
Solving this equation we obtain

$$K_{min} = M \cdot \left(\varepsilon \times \gamma^{\frac{(1-\delta)\gamma}{\varepsilon}} \times \delta^{\frac{-\delta\gamma}{\varepsilon}} \times (2+\beta)^{\frac{1}{\varepsilon}} \times \left(\frac{p}{r}\right)^{\frac{1-\gamma}{\varepsilon}} \times T^{\frac{\delta\gamma}{\varepsilon}} - T\right),$$

where $\varepsilon = 1 - \gamma + \delta\gamma$.



Determination of the separating capital level



Separating capital level is linearly proportional to income

In order to provide an illustration we set numerical values representing a real-life situation:

- $M = 240000$ rubles a year (that is 20000 rubles a month per adult, so the family is relatively poor);
- $p = 40000$ rubles per square meter;
- $r = 6000$ rubles a year per square meter;
- $T = 25$ years;
- $\beta = \gamma = \delta = 0.5$.

Plugging these values in the formula above results in the separating capital level of approximately 326000 rubles. Since actual maternity capital offered by the government exceeds this figure (and equals $K = 450000$ rubles), the couple under consideration will choose to have a child as this will bring them higher decision utility $TU^*_K(K)$.

5.3 Effects of Variables on Separating Capital Level

- The separating capital level is *linearly proportional* to income M. The poorer the family the less capital government needs to motivate it to have a child. This explains why maternity capital is a strong incentive to give birth for poor families but not for rich ones.

$$M \uparrow \quad \Rightarrow \quad K_{min} \uparrow \text{ by the same factor}$$

- If the share of child's consumption β rises, the separating capital level also increases, because the family has to spend more money on consumption:

$$\beta \uparrow \quad \Rightarrow \quad (2+\beta)^{\frac{1}{\varepsilon}} \uparrow \quad \Rightarrow \quad K_{min} \uparrow$$

- If the annual rent per square meter r goes down, then living in a rented apartment becomes cheaper. Also, when the price per square meter p goes up, buying a flat becomes more expensive. Both changes make staying at the rented apartment more appealing, that means in order to motivate the family to have a child the capital has to be larger.

$$\left[\begin{matrix} r \downarrow \\ p \uparrow \end{matrix}\right. \quad \Rightarrow \quad \left(\frac{p}{r}\right)^{\frac{1-\gamma}{\varepsilon}} \uparrow \quad \Rightarrow \quad K_{min} \uparrow$$

- The overall price level $\mathbb{P}$ characterizing the city where the family lives has no effect on the separating capital level due to the specifics of the model. The price level can be extracted from the selected Cobb-Douglas utility function in a term $-\gamma \cdot ln(\mathbb{P})$, that is why an increase in prices will change total decision utility by the same number both in the "no K" case and in the "K" case, thus producing no impact on the couple's choice.

$$\mathbb{P} \uparrow \quad \Rightarrow \quad K_{min} = const$$

- An effect of the mortgage tenure T on the separating capital level is less transparent. From the solution of the UMP for the "K" case it follows that optimal consumption is a decreasing hyperbolic function of T, while optimal apartment area is an increasing linear function of T:

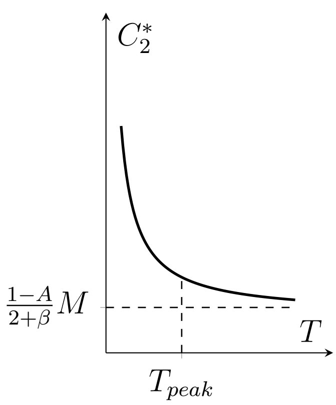
$$\frac{\partial C_2^*}{\partial T} = \frac{1-A}{2+\beta} \times \frac{-K}{T^2} < 0, \qquad \frac{\partial F_1^*}{\partial T} = \frac{AM}{p} = const > 0.$$

If T is increased by one year then at low values of T (assuming δ is not high) consumption demanded falls significanty, while at high values of T the fall in consumption is quite small. On the other hand, a one-year increase in T rises apartment area demanded by a constant number of squares. Therefore, if T is below some T_{peak} and is increased by one year then decision utility corresponding to the "K" case falls making the separating capital level larger. Since for high (above T_{peak}) values of T the opposite happens, we infer that relation of separating capital level to mortgage tenure is non-monotonic.

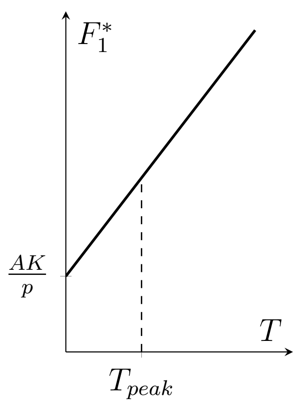
$$\begin{cases} T_{peak} - T \geq 1 \text{ year} \\ T \uparrow \text{ by 1 year} \end{cases} \Rightarrow \begin{cases} C_2^* \downarrow\downarrow\downarrow \\ F_1^* \uparrow\uparrow \end{cases} \Rightarrow \forall K \; TU_K^*(K) \downarrow \;\Rightarrow\; K_{min} \uparrow$$

$$\begin{cases} T \geq T_{peak} \\ T \uparrow \text{ by 1 year} \end{cases} \Rightarrow \begin{cases} C_2^* \downarrow \\ F_1^* \uparrow\uparrow \end{cases} \Rightarrow \forall K \; TU_K^*(K) \uparrow \;\Rightarrow\; K_{min} \downarrow$$

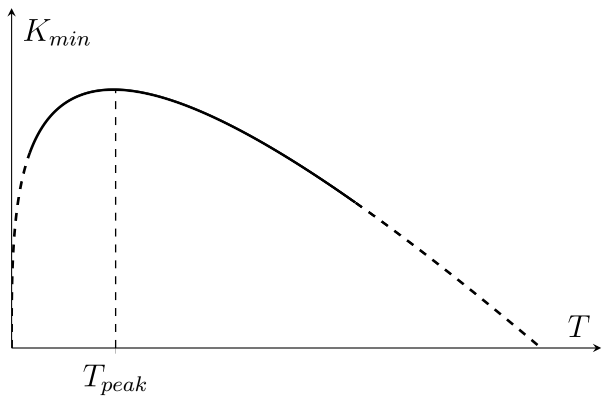
A practical consequence is that if banks offer long mortgages (that is usually the case) and tend to increase mortgage tenures further then the government is able to stimulate the same number of births offering smaller maternity capital.



Consumption
demanded of T



Flat area
demanded of T



Separating capital level
as a function of mortgage tenure

5.4 Maternity Capital Decreases Welfare

The proposition

Accurately the phenomenon is formulated as follows: *under any reasonable values of the discount factor δ and mortgage tenure T the decision to give birth and receive maternity capital decreases the aggregate utility ΣU experienced by each parent during the first $(1+T)$ years.*

Before justifying this proposition it is necessary to define the aggregate experienced utility. It is a regular (non-discounted) sum of all utilities, received by an adult during the considered time period.

Here are the expressions for aggregate utility experienced over $(1+T)$ years in both "no K" and "K" cases:

$$\Sigma U_{noK} = (1+T)u_0^*,$$
$$\Sigma U_K(K) = u_1^* + Tu_2^*,$$

where $u_0^* = u\left(\frac{C_0^*}{\mathbb{P}}, \frac{F_0^*}{2}\right)$, $u_1^* = u\left(\frac{C_1}{\mathbb{P}}, \frac{F_1^*}{2+\beta}\right)$ and $u_2^* = u\left(\frac{C_2^*}{\mathbb{P}}, \frac{F_1^*}{2+\beta}\right)$.

Thus, the proposition states that the following system of inequalities holds under any reasonable pair (δ, T):

$$\begin{cases} TU_{noK}^* < TU_K^*(K) \\ \Sigma U_{noK} > \Sigma U_K(K) \end{cases}$$

The first inequality means that the family rationally chooses to have a baby and to receive the maternity capital, while the second inequality states that this decision makes each adult worse off in terms of aggregate experienced utility.

The proof

The proof starts with rewritting the inequalities in terms of optimized utility values u_0^*, u_1^*, u_2^*:

$$\begin{cases} \frac{1}{1-\delta}u_0^* < u_1^* + \frac{\delta}{1-\delta}u_2^* \\ (1+T)u_0^* > u_1^* + Tu_2^* \end{cases} \Leftrightarrow \begin{cases} u_0^* < (1-\delta)u_1^* + \delta u_2^* \\ u_0^* > \frac{1}{1+T}u_1^* + \frac{T}{1+T}u_2^* \end{cases}$$

Therefore, the following inequality is obtained:

$$\frac{1}{1+T}u_1^* + \frac{T}{1+T}u_2^* < (1-\delta)u_1^* + \delta u_2^*. \qquad (\star)$$

Note that both in the LHS and in the RHS weights of the optimized utility values sum up to 1. Recall that $u_1^* = u\left(\frac{C_1}{\mathbb{P}}, \frac{F_1^*}{2+\beta}\right)$ and $u_2^* = u\left(\frac{C_2^*}{\mathbb{P}}, \frac{F_1^*}{2+\beta}\right)$. Thus,

$$u_1^* > u_2^* \quad \Leftrightarrow \quad C_1 > C_2^* \quad \Leftrightarrow \quad \frac{M}{2+\beta} > \frac{\delta\gamma}{1-\gamma+\delta\gamma} \times \frac{MT+K}{(2+\beta)T} \quad \Leftrightarrow \quad \frac{MT}{K} > \frac{\delta\gamma}{1-\gamma}.$$

If γ is not extremely close to 1 then the last inequality would hold because $\delta < 1$ and $MT/K > 1$ (usually capital is less than total income earned by husband in T years).
Now assuming that $u_1^* > u_2^*$, the $(\star)$ inequality holds if and only if the weight of u_1^* in the LHS is lower:

$$\frac{1}{1+T} < 1-\delta \quad \Leftrightarrow \quad \delta < \frac{T}{1+T}.$$

Typically, mortgage tenure T is long (about 10-30 years), so, the fraction $\frac{T}{1+T}$ is almost 1. Thus, under any reasonable values of δ and T the system solved above holds meaning that parents who rationally decide to have a child and obtain the maternity capital are worse off with their decision in terms of aggregate utility received during the $1+T$ years after giving birth to the child. This happens because evaluating the "K" case the couple does not discount high utility of year 1 but discounts low utility of all the subsequent years and thus chooses to give birth.

6 Conclusion

To sum up, the derived model approaches the phenomenon of the Russian family support system from the microeconomic perspective.

A rational family of a man and a woman trying to decide whether to have a child or not is considered. Therefore, two scenarios are regarded: with maternity capital (a child) and without it (no children). Via utility maximization, optimal distributions between consumption and apartment costs for both cases are found.

Then the separating capital level is derived to show starting from which amount of maternity capital the rational family decides to give birth to a baby. It turns out that while the share of child's consumption and price per square have a positive effect on the separating capital level, the rent per square affects it negatively. The level of prices has no influence on the separating amount because of the chosen Cobb-Douglas preferences. The income level demonstrates a linear proportionality showing that capital motivates poor families but not rich ones thus increasing the general poverty level in the country. Surprisingly, relation of the separating capital level to the mortgage tenure is non-monotonic.

In the last part of the paper, it was proved that under the assumptions made a rational choice in favour of having a baby decreases the couple's welfare. Thus, the model demonstrates inefficiency of the current maternity capital policy due to the fact that it provokes a decrease in consumption and welfare of a household (poverty goes up). For many low-income families maternity capital is an incentive to give birth, however it is a mistake to treat such social support as a way to escape poverty since it has an opposite effect on family welfare.

The study can be enhanced by relaxing unrealistic assumptions made for simplicity that can be a subject for further research.