Expectation (definition)

$$\mathrm{E}X = \sum_{i=1}^{n} x_i P(X = x_i) = \sum_{i=1}^{n} X(\omega_i) P(\omega_i)$$

Properties of expectation

$$\mathrm{E}(a) = a \qquad \mathrm{E}(aX) = a\mathrm{E}X \qquad \mathrm{E}(X+Y) = \mathrm{E}X + \mathrm{E}Y$$

Variance (definition)

$$\mathrm{Var}\, X = \mathrm{E}((X - \mathrm{E}X)^2)$$

Variance (convenient formula)

$$\boxed{\mathbf{Var}\, \boldsymbol{X} = \mathbf{E}(\boldsymbol{X}^2) - (\mathbf{E}\boldsymbol{X})^2}$$

Properties of variance

$$\mathrm{Var}(a) = 0 \qquad \mathrm{Var}(aX + b) = a^2\, \mathrm{Var}\, X$$

Standard deviation (definition)

$$\boxed{\boldsymbol{\sigma X} = \sqrt{\mathbf{Var}\, \boldsymbol{X}}}$$

Properties of standard deviation

$$\sigma(a) = 0 \qquad \sigma(aX + b) = |a|\sigma X$$

Independent random variables (definition)

$$P\big((X = x_i) \cap (Y = y_j)\big) = P(X = x_i) \cdot P\big(Y = y_j\big) \text{ for all } (x_i, y_j)$$

Properties of independent random variables

$$\mathrm{E}(XY) = \mathrm{E}X \cdot \mathrm{E}Y$$

$$\mathrm{Var}(X+Y) = \mathrm{Var}\, X + \mathrm{Var}\, Y \qquad \sigma(X+Y) = \sqrt{(\sigma X)^2 + (\sigma Y)^2}$$

Uniform distribution on the set $S = \{\mathrm{a}, \mathrm{a}+1, \dots, \mathrm{b}\}$ where $P(X = k) = \frac{1}{b-a+1}$ for all $k \in S$

$$\mathrm{E}X = \frac{a+b}{2} \qquad \mathrm{Var}\, X = \frac{(b-a+1)^2 - 1}{12}$$

Bernoulli distribution with probability of success p

$$\mathrm{E}X = p \qquad \mathrm{Var}\, X = p(1-p)$$

Binomial distribution with parameters (n, p) where $P(X = k) = C_n^k p^k (1-p)^{n-k}$

$$\mathrm{E}X = np \qquad \mathrm{Var}\, X = np(1-p)$$

Clarification. A sum of n identical Bernoulli random variables is distributed binomially.

Marginal distribution in *a joint distribution*

$$P(X = x_i) = \sum_{j=1}^{m} P\big((X = x_i) \cap (Y = y_j)\big)$$

Conditional expectation (definition)

$$\mathrm{E}(X|A) = \sum_{i=1}^{n} x_i P(X = x_i | A)$$

Covariance (definition)

$$\operatorname{cov}(X,Y) = \mathrm{E}((X - \mathrm{E}X)(Y - \mathrm{E}Y))$$

Covariance (convenient formula)

$$\boxed{\mathbf{cov}(\boldsymbol{X},\boldsymbol{Y}) = \mathbf{E}(\boldsymbol{XY}) - \mathbf{E}\boldsymbol{X} \cdot \mathbf{E}\boldsymbol{Y}}$$

Properties of covariance

$$\operatorname{cov}(X,Y) = \operatorname{cov}(Y,X) \qquad \operatorname{cov}(X,a) = 0 \qquad \operatorname{cov}(X,X) = \operatorname{Var} X$$

$$\operatorname{cov}(X,Y+a) = \operatorname{cov}(X,Y) \qquad \operatorname{cov}(aX+bY,Z) = a\operatorname{cov}(X,Z) + b\operatorname{cov}(Y,Z)$$

Variance of a sum

$$\boxed{\mathbf{Var}(\boldsymbol{X}+\boldsymbol{Y}) = \mathbf{Var}\,\boldsymbol{X} + \mathbf{2}\,\mathbf{cov}(\boldsymbol{X},\boldsymbol{Y}) + \mathbf{Var}\,\boldsymbol{Y}}$$

Uncorrelated random variables (definition)

$$\operatorname{cov}(X,Y) = 0$$

Equivalent definitions of uncorrelated random variables

$$\mathrm{E}(XY) = \mathrm{E}X \cdot \mathrm{E}Y \qquad \operatorname{Var}(X+Y) = \operatorname{Var} X + \operatorname{Var} Y$$

Some true statements about random variables and random *events*

Two random variables are *independent*. $\Rightarrow$ These variables are *uncorrelated*.

Two random variables are *uncorrelated*. $\nRightarrow$ These variables are *independent*.

Two random events are *independent*. $\nRightarrow$ These events are *disjoint*.

Two random events are *disjoint*. $\nRightarrow$ These events are *independent*.

Clarification. Definition of independent events: $P(A \cap B) = P(A) \cdot P(B)$. Definition of disjoint events: $P(A \cap B) = 0$

Correlation coefficient (definition)

$$\rho(X,Y) = \operatorname{cov}\left(\frac{X - \mathrm{E}X}{\sigma X}, \frac{Y - \mathrm{E}Y}{\sigma Y}\right)$$

Correlation coefficient (convenient formula)

$$\boxed{\boldsymbol{\rho}(\boldsymbol{X},\boldsymbol{Y}) = \frac{\mathbf{cov}(\boldsymbol{X},\boldsymbol{Y})}{\boldsymbol{\sigma X} \cdot \boldsymbol{\sigma Y}}}$$

Interpretation of the value of the correlation coefficient

Exactly −1	A *perfect* negative linear relationship
-0.7	A *strong* negative linear relationship
-0.5	A *moderate* negative linear relationship
-0.3	A *weak* negative linear relationship
0	No linear relationship
+0.3	A *weak* positive linear relationship
+0.5	A *moderate* positive linear relationship
+0.7	A *strong* positive linear relationship
Exactly +1	A *perfect* positive linear relationship